Spiral plat avec une seule courbe terminale externe

Déformée élastique en position horizontale

Cas d'une montre bracelet

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.03\,mm \quad ha = 0.15\,mm \quad S = 4.5\times10^{-3}\,mm^2 \quad TOL := 10^{-12}$

$d2_{sp} = 4.52\,mm \quad d1_{sp} = 1.1\,mm \quad p_{sp} = 0.135\,mm \quad n_{sp} = 12.667$

$L := L_{sp} \quad L = 11.182\,cm \quad \psi_0 := 2\cdot\pi\cdot n_{sp} \quad \psi_0 = 4.56\times10^3\,deg$

Position du point de raccordement sur le spiral $\quad \alpha_A := \pi \quad r_A := 0.5\cdot d2_{sp} \quad z_A := r_A\cdot e^{i\cdot\alpha_A}$

Forme initiale du spiral

$$a := \frac{p_{sp}}{2\cdot\pi} \quad r_s(\alpha) := r_A - a\cdot(\alpha-\alpha_A) \quad x_{0s}(\alpha) := r_s(\alpha)\cdot cos(\alpha) \quad y_{0s}(\alpha) := r_s(\alpha)\cdot sin(\alpha)$$

$$s(\alpha) := \frac{1}{2\cdot a}\cdot\left(r_A^{\,2} - r_s(\alpha)^2\right) \quad s(\alpha) := r_A\cdot(\alpha-\alpha_A) - \frac{a}{2}\cdot(\alpha-\alpha_A)^2$$

Courbe terminale externe

$$r_{t1} := 0.8 \quad r_{t1} := racine\left[(2\cdot r_{t1}-1)^4 - 4\cdot(1-r_{t1})^4 - \pi^2\cdot r_{t1}^{\,2}\cdot(1-r_{t1})^2, r_{t1}\right]\cdot r_A \quad r_{t1} = 0.832\,r_A$$

$$r_{t2} := 2\cdot r_{t1} - r_A \quad r_{t2} = 0.665\,r_A \quad \beta_0 := arctan\left[\frac{\pi\cdot r_{t1}}{2\cdot(r_A - r_{t1})}\right] \quad \beta_0 = 82.695\,deg \quad l_t := r_{t2}\cdot\beta_0 + \pi\cdot r_{t1}$$

$$x_{0t1}(\alpha_t) := -r_A + r_{t1}\cdot(1+cos(\alpha_t)) \quad y_{0t1}(\alpha_t) := r_{t1}\cdot sin(\alpha_t)$$

$$x_{0t2}(\beta_t) := r_{t2}\cdot cos(\beta_t) \quad y_{0t2}(\beta_t) := r_{t2}\cdot sin(\beta_t)$$

Position des goupilles de raquettes $\quad r_{GR} := r_{t2} \quad \alpha_{GR} := -\beta_0 \quad \alpha_{GR} = -82.695\,deg$

$x_{GR} := x_{0t2}(\alpha_{GR}) \quad y_{GR} := y_{0t2}(\alpha_{GR})$

Position du point d'attache à la virole $\quad r_V := 0.5\cdot d1_{sp} \quad \alpha_V(\theta) := \alpha_A + \psi_0 + \theta$

$x_V(\theta) := r_V\cdot cos(\alpha_V(\theta)) \quad y_V(\theta) := r_V\cdot sin(\alpha_V(\theta)) \quad L_t := L + l_t$

Amplitude stationnaire du balancier $\quad \theta_0 = 270\,deg$

Contrainte maximum

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$$I_{33} := I_{f\_rect}(ép, ha) \quad W_{f3} := W_{f\_rect}(ép, ha) \quad \sigma_{max} := \frac{E\cdot I_{33}}{L\cdot W_{f3}}\cdot\theta_0 \quad \sigma_{max} = 132.293\,N\cdot mm^{-2}$$

Centres de masse

Partie spiralée $\quad z_{0s}(\alpha) := x_{0s}(\alpha) + i\cdot y_{0s}(\alpha)$

$$\zeta_{0s} := \frac{1}{L}\cdot\int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha \quad \xi_{0s} := Re(\zeta_{0s}) \quad \eta_{0s} := Im(\zeta_{0s})$$

$$\xi_{0s} = 1.361\times10^{-3}\,mm \quad \eta_{0s} = -0.047\,mm$$

Courbe terminale externe

$z_{0t1}(\alpha_t) := x_{0t1}(\alpha_t) + i\cdot y_{0t1}(\alpha_t)$ $\qquad z_{0t2}(\beta_t) := x_{0t2}(\beta_t) + i\cdot y_{0t2}(\beta_t)$

$$\zeta_{0t} := \frac{1}{l_t}\cdot\left(\int_0^{\pi} z_{0t1}(\alpha_t)\cdot r_{t1}\, d\alpha_t + \int_{-\beta_0}^{0} z_{0t2}(\beta_t)\cdot r_{t2}\, d\beta_t\right)$$

$\xi_{0t} := Re(\zeta_{0t})$ $\qquad \eta_{0t} := Im(\zeta_{0t})$ $\qquad \xi_{0t} = 0\,mm$ $\qquad \eta_{0t} = 0.632\,mm$

Centre de masse du spiral $\qquad \zeta_s := \frac{1}{L_t}\cdot(L\cdot\zeta_{0s} + l_t\cdot\zeta_{0t})$ $\qquad \zeta_s = 1.27\times 10^{-3} - 1.421i\times 10^{-3}\,mm$

Première approximation de la déformée du spiral

Courbe terminale externe

$\varphi_{0t2}(\beta_t) := \frac{\pi}{2} + \beta_t$ $\quad z_{GR} := x_{GR} + i\cdot y_{GR}$ $\quad z_{1t2}(\theta,\beta_t) := z_{GR} + r_{t2}\cdot\int_{-\beta_0}^{\beta_t} i\cdot e^{i\cdot\beta'_t}\cdot exp\left[i\cdot\frac{\theta}{L_t}\cdot\left[r_{t2}\cdot(\beta_0 + \beta'_t)\right]\right] d\beta'_t$

$z_{1t2}(\theta,\beta_t) := z_{GR} + \frac{L_t\cdot r_{t2}}{L_t + \theta\cdot r_{t2}}\cdot\left[exp\left[i\cdot\frac{\beta_t\cdot L_t + \theta\cdot r_{t2}\cdot(\beta_0 + \beta_t)}{L_t}\right] - exp(-i\cdot\beta_0)\right]$ $\qquad z_{1C}(\theta) := z_{1t2}(\theta, 0)$

$\Delta\varphi_{1C}(\theta) := \frac{\theta}{L_t}\cdot r_{t2}\cdot\beta_0$ $\qquad \Delta\varphi_{1C}(\theta_0) = 4.883\,deg$

$\varphi_{0t1}(\alpha_t) := \alpha_t + \frac{\pi}{2}$ $\qquad \Delta z_{1t1}(\theta,\alpha_t) := r_{t1}\cdot\int_0^{\alpha_t} i\cdot e^{i\cdot\alpha'_t}\cdot exp\left(i\cdot\frac{\theta}{L_t}\cdot r_{t1}\cdot\alpha'_t\right) d\alpha'_t$

$\Delta z_{1t1}(\theta,\alpha_t) := \frac{L_t\cdot r_{t1}}{L_t + \theta\cdot r_{t1}}\cdot\left(exp\left(i\cdot\alpha_t\cdot\frac{L_t + \theta\cdot r_{t1}}{L_t}\right) - 1\right)$ $\qquad z_{1t1}(\theta,\alpha_t) := z_{1C}(\theta) + \Delta z_{1t1}(\theta,\alpha_t)\cdot e^{i\cdot(\Delta\varphi_{1C}(\theta))}$

$\Delta\varphi_{1A}(\theta) := \theta\cdot\frac{l_t}{L_t}$ $\qquad \Delta\varphi_{1A}(\theta_0) = 18.192\,deg$ $\qquad z_{1A}(\theta) := z_{1t1}(\theta,\pi)$

Partie spiralée

$s'(\alpha) := r_A - a\cdot(\alpha - \alpha_A)$ $\qquad z'_0(\alpha) := \left[-a + i\cdot\left[r_A - a\cdot(\alpha - \alpha_A)\right]\right]\cdot exp(i\cdot\alpha)$

$\Delta z_{1s}(\theta,\alpha) := \int_{\alpha_A}^{\alpha} z'_0(\alpha')\cdot exp\left(i\cdot\theta\cdot\frac{s(\alpha')}{L_t}\right) d\alpha'$ $\qquad z_{1s}(\theta,\alpha) := z_{1A}(\theta) + \Delta z_{1s}(\theta,\alpha)\cdot e^{i\cdot\Delta\varphi_{1A}(\theta)}$

Graphe de la déformation

Forme naturelle

$n_t := 201$ $\quad j := 0..n_t - 1$ $\quad \Delta\alpha_t := \frac{\pi}{n_t - 1}$ $\quad \alpha_{t_j} := j\cdot\Delta\alpha_t$ $\quad x_{t1_j} := x_{0t1}(\alpha_{t_j})$ $\quad y_{t1_j} := y_{0t1}(\alpha_{t_j})$

$\Delta\beta_t := \frac{\beta_0}{n_t - 1}$ $\quad \beta_{t_j} := j\cdot\Delta\beta_t - \beta_0$ $\quad x_{t2_j} := x_{0t2}(\beta_{t_j})$ $\quad y_{t2_j} := y_{0t2}(\beta_{t_j})$ $\quad x_t := pile(x_{t2}, x_{t1})$ $\quad y_t := pile(y_{t2}, y_{t1})$

$n := 50\cdot partentière(n_{sp}) + 1$ $\quad i := 0..n - 1$ $\quad \Delta\alpha := \frac{\psi_0}{n - 1}$ $\quad \alpha_i := \pi + i\cdot\Delta\alpha$

$x_{s_i} := x_{0s}(\alpha_i)$ $\quad y_{s_i} := y_{0s}(\alpha_i)$ $\quad x_0 := pile(x_t, x_s)$ $\quad y_0 := pile(y_t, y_s)$ $\quad mod(\psi_0 + \pi, 2\cdot\pi) = 60\,deg$

$r_0 := \overrightarrow{\sqrt{x_0^{\,2} + y_0^{\,2}}}$ $\quad \beta_s := \overrightarrow{Atan(x_0, y_0)}$ $\quad n_{pt} := dernier(\beta_s)$ $\quad \beta_{s_{npt}} = 60\,deg$

$mod(\alpha_V(0), 2\cdot\pi) = 60\,deg$

Déformée

$z_{td2} := \overrightarrow{z_{1t2}(\theta_0, \beta_t)}$ $\quad z_d := z_{td2}$ $\quad z_{td1} := \overrightarrow{z_{1t1}(\theta_0, \alpha_t)}$ $\quad z_d := pile(z_{td2}, z_{td1})$

$z_{sd} := \overrightarrow{z_{1s}(\theta_0, \alpha)}$ $\quad z_d := pile(z_d, z_{sd})$

$n_{pt} := dernier(z_d)$ $\quad x_d := Re(z_d)$ $\quad y_d := Im(z_d)$ $\quad r_d := \overrightarrow{(|z_d|)}$ $\quad r_{d_{npt}} = 0.53\,mm$

$\beta_d := \overrightarrow{Atan(x_d, y_d)}$ $\quad \beta_{d_0} = 277.305\,deg$ $\quad \beta_{d_{npt}} = 328.628\,deg$ $\quad mod(\alpha_V(\theta_0), 2\cdot\pi) = 330\,deg$

$r_{GR} = 1.502\,mm$ $\quad r_V = 0.55\,mm$ $\quad x_V(\theta_0) = 0.476\,mm$ $\quad y_V(\theta_0) = -0.275\,mm$

$x_{d_{npt}} - x_V(\theta_0) = -0.024\,mm$ $\quad y_{d_{npt}} - y_V(\theta_0) = -7.91 \times 10^{-4}\,mm$